SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Show that

$$\log\sqrt{12} = 1 + \left(\frac{1}{2}+\frac{1}{3}\right)\frac{1}{4} + \left(\frac{1}{4}+\frac{1}{5}\right)\frac{1}{4^2} + \left(\frac{1}{6}+\frac{1}{7}\right)\frac{1}{4^3} + \ldots$$

17. Solve the equation $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$.

18. Show that if $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$, then A satisfies the equation $A^3 - 3A^2 + 3A - 2I = 0$.

19. Find the inverse of the matrix $A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 0 & 5 \\ -1 & 1 & 6 \end{bmatrix}$.

20. Express $\sin^7\theta\cos^3\theta$ as the sum of sines of multiple of θ.

NOVEMBER/DECEMBER 2024

FAEL12/CAEL12 — BASIC MATHEMATICS – I

Time : Three hours Maximum : 75 marks

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SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Write the expansion of $(1-x)^{-n}$.

2. Resolve $\frac{2x+2}{(x-1)(x-2)}$ into partial fractions.

3. Change the equation $2x^4 - 3x^3 + 3x^2 - x + 2 = 0$ into another the coefficient of whose highest term will be unity.

4. Find the quotient and remainder when $3x^3 + 8x^2 + 8x + 12$ is divided by $x - 4$.

5. State Cayley Hamilton's theorem.

6. Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 7 & -2 \\ -2 & 1 \end{bmatrix}$.

7. If $A = \begin{bmatrix} 7 & 4 \\ -1 & 0 \end{bmatrix}$ then find A^{-1}.

8. Find the value of x, y and z $\begin{bmatrix} x+3 & 2y \\ z+4 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 8 & 1 \end{bmatrix}$.

9. If $x = \cos\theta + i\sin\theta$ then find $x^n + \frac{1}{x^n}$ and $x^n - \frac{1}{x^n}$.

10. Write the expansion of $\tan n\theta$.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Resolve into partial fractions $\frac{7x-1}{1-5x+6x^2}$.

Or

(b) Find the sum of infinity of the series $\frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{6.12.18} + \ldots$.

12. (a) Increase by 7 the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$.

Or

(b) Solve the equation $x^4 - 2x^3 - 13x^2 + 38x + 24 = 0$ by finding the rational roots by Newtons method.

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13. (a) Show that $A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{-2}{3} & \frac{2}{3} & \frac{-1}{3} \end{bmatrix}$ is orthogonal.

Or

(b) Find the characteristics equation of the matrix and hence find the inverse $A = \begin{bmatrix} 5 & 4 \\ 1 & 3 \end{bmatrix}$.

14. (a) Solve the equations

$6x + 2y - 2z = 6$

$-2x + 2y + 2z = 2$

$2x + 2y + 2z = 6$

Or

(b) Verify the result $(AB)C = A(BC)$, when $A = \begin{bmatrix} 1 & -1 \\ 4 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $C = \begin{bmatrix} 1 & -1 \\ 4 & 0 \end{bmatrix}$.

15. (a) Expand $\sin^7\theta$ in a series of sines of multiple of θ.

Or

(b) Write down the expansion of $\cos 9\theta$.